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10397/NH

CS/2110

MATHEMATICAL METHODS-I

Paper-III(i)

Semester-V (Dec-19)

Time allowed : 3 Hours] [Maximum Marks : 40

Note : Candidates are required to attempt two questions each from Section A and B. Entire Section C is compulsory.

SECTION-A

1. Find the Fourier transform of : 6

$$f(t) = \begin{cases} t & -a \quad t \quad a \\ 0 & t < -a \text{ or } t > a \end{cases}$$

2. Expand f(x) = x; 0 < x < 2, in a half range (a) sine series, (b) cosine series. 6

3. (i) Assuming that the Fourier series corresponding to f(x) converges uniformly to f(x) in (–L, L), prove that : 3

$$\frac{1}{L}\int_{-L}^{L} \{f(x)\}^2 dx = \frac{a_0^2}{2} + \quad (a_n^2 + b_n^2)$$

(ii) Show that an even function can have no sine terms in its Fourier expansion. 3

4. Expand f(x) = x^2 ; 0 < x <   in a Fourier series is period of 2 . Also deduce that : 6

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} \cdots\cdots = \frac{\ ^2}{6}$$

SECTION-B

5. State and prove Convolution theorem for Inverse Laplace Transforms. 6

6. State and prove Heaviside's expansion formula and use it to find the Laplace inverse of : 6

$$\frac{2s + 1}{s(s - 1)(s + 1)}$$

7. (i) Evaluate the Laplace transformation for : 3

$f(t) = e^{2t} t^2 \sin 4t$

(ii) Find the Inverse Laplace transformation of :

$$F(s) = \frac{1}{s^3(s + 1)}$$ 3

8. (i) Apply Convolution theorem to show that : 3

$$\int_0^t \sinh z \cosh (t - z)\, dz = \frac{1}{2} t \sinh t$$

(ii) Prove that $1 * 1 * 1 \ldots\ldots * 1$ (k times) $= \dfrac{t^{k-1}}{(k-1)!}$

where * denotes the convolution. 3

SECTION-C

9. (i) Find the Laplace inverse of $F(s) = \dfrac{1}{s^2 + 2s + 1}$

(ii) Write the formula for $L[f''(t)]$

(iii) Find the Laplace transformation of $f(t)=t^2 \sin t$.

(iv) State Drichlet's conditions.

(v) Classify that the given function is even or odd

$$f(x) = \begin{cases} 2 & 0 < x < 3 \\ -2 & -3 < x < 0 \end{cases}$$

(vi) Define Periodic function.

(vii) Define function of exponential order

(viii) State the relation between Beta and Gamma functions. 8×2=16