D-27/2110

MATHEMATICAL FOUNDATION OF STATISTICS -105

SEMESTER-I

TIME ALLOWED 3 Hrs

M.M 70

NOTE: The candidates are required to attempt two questions each from Section A & B Section C will be compulsory

Section A

1. Two fair dice are thrown independently. Three events A, B and C is defined as follows:

A: Even face with first dice.

B: Even face with second dice.

C: Sum of the points on the two dice is odd.

Discuss the independence of events A, B and C.

2. (a) A continuous r.v. X has the following probability density function: $f(x) = A + Bx,\ 0 \le x \le 1$ If the mean of the distribution is ½ . Find the value of A and B.

(b) The probability of bomb hitting a target is 1/5. Two bombs are enough to destroy a bridge. If six bombs are aimed at the bridge, find the probability that the bridge is destroyed.

3. (a) The length of time a person speaks over phone follows an exponential distribution with mean 6. What is the probability that the person will talk for (i) more than 8 minutes and (ii) between 4 and 8 minutes?

(b) Find the value of c such that $P(|X-25|<c)=0.9544$ where $X \sim N(25, 36)$. Given that $P(Z<-2) = 0.0228$ and $P(Z<-1.69) = 0.0456$, Z being a standard normal variate.

4. A random variable X has a density function $f(x) = \begin{cases} \lambda e^{-\lambda x} & ; x > 0 \\ 0 & ; elsewhere \end{cases}$. Find the moment generating function and the mean and variance.

SECTION B

5. (a) State and Prove Bayes theorem.

(b) In a bolt factory, machines A, B, C manufacture respectively 25%, 35% and 40% of the total. Of their output 5, 4, 2 percent are known to be defective bolts. A bolt is drawn at random from the product and is found to be defective. What are the probabilities that it was manufactured by machine B or C.

6. Let $\{X_n\}$ be a sequence of mutually independent random variables such that $X_n = \pm 1$ with probability $\frac{1-2^{-n}}{2}$ and $X_n = \pm 2^{-n}$ with probability 2^{-n-1}. Examine whether the weak law of large numbers can be applied to the sequence $\{X_n\}$.

7. (a) The joint probability mass function of X and Y is defined as $P(X=0, Y=-1) = 1/8$, $P(X=0, Y=1) = 3/8$, $P(X=1, Y=-1) = 2/8$ and $P(X=1, Y=1) = 2/8$. Find the correlation coefficient of (X, Y).

P.T.O.

(b) A random sample of size 100 is taken from a population whose mean is 80 and variance is 400. Using central limit theorem, with what probability can we assert that the mean of the sample will not differ from $\mu = 80$ by more than 6?

8. Two dimensional r.v. (X,Y) have the joint density $f(x,y) = \begin{cases} 8xy & , 0 < x < y < 1 \\ 0 & , otherwise \end{cases}$.

Find (i) $P(X < \frac{1}{2} \cap Y < \frac{1}{4})$ (ii) find the marginal and conditional distributions. (iii) Are X and Y are independent.

SECTION C

9. Write in brief.

a) State the axiomatic definition of probability.

b) Define probability and conditional probability for an event A of a sample space.

c) For two events A and B such that P(A) = ¼, P(B)= 1/3 and P(AUB)=1/2. Find P(B|A).

d) State moment generating function.

e) A random variable X has the following probability distribution:

Value of X:	0	1	2	3	4	5	6	7	8
p(x)	k	3k	5k	7k	9k	11k	13k	15k	17k

Determine the value of k.

f) In Poisson frequency distribution, frequency corresponding to 3 successes is $\frac{2}{3}$ times frequency corresponding to 4 successes. Find the variance of the distribution.

g) If the m.g.f. of X is given by $M_X(t) = \frac{1}{1-t^2}$ where $|t| < 1$. Find the moment generating function of $Y = \frac{X-4}{4}$.

h) Differentiate between the discrete and continuous random variables.

i) State weak law of large numbers

j) If X and Y be two independent random variables then find the moment generating function of $X + Y$.

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