8402/NH

A-2110

DIFFERENTIAL EQUATIONS-II

SEMESTER-I

TIME 3 HOURS MM: 40

Note :The candidates are required to attempt two questions each from Section A & B ,Section C will be compulsory.

Section A (6X2=12)

Q1 i) Solve $y - x\frac{dy}{dx} = a(y^2 + \frac{dy}{dx})$

ii) Solve $(2\sqrt{xy} - x)dy + ydx = 0$

Q2 i) Solve $y \sin 2x\, dx - (1 + y^2 + cos^2x)dy = 0$

ii) Solve $\frac{dy}{dx} + xsin\, 2y = x^3cos^2y$

Q3 Solve $(D^2 + a^2)y = \cos ax$

Q4 Use method of variation of parameters to solve $y'' + y = \frac{1}{1+\sin x}$

Section B (6X2=12)

Q5 Solve $[(1 + 2x)^2D^2 - 6(1 + 2x)D + 16]y = 8(1 + 2x)^2$

Q6. Find the general solution of $y'' + (x - 3)y' + y = 0$ near x=2.

Q7 Show that $(n + 1)P_n = P'_{n+1} - xP'_n$

Q8 Show that $J_n(x) = \frac{1}{\pi}\int_0^{\pi} \cos(n\varphi - xsin\varphi)\, d\varphi$, where n is positive integer.

Section C (8X2=16)

Q9 i) Find the differential equation of all circles tangent to y axis.

ii) Define Wronskian. Show that the vectors e^{2x} and e^{3x} are linearly independent vectors.

iii) Check whether the given differential equation is exact or not and hence solve

$$ydx - xdy + (1 + x^2)dx + x^2sinydy = 0$$

iv) Integrate $(1 + x^2)\frac{dy}{dx} + 2xy - 4x^2 = 0.$

v) State Rodrigue's formula

vi) Solve : $(x^2D^2 + xD - 4)y = 0$

vii) Determine whether x=0 is an ordinary point or regular singular point for the differential equation

$$2x^2y'' - xy' + (x - 5)y = 0$$

viii) For Bessel's function $J_n(x)$, find out a and b, where $\frac{d}{dx}J_n(x) = aJ_{n-1}(x) + bJ_{n+1}(x)$