8401/NH

A--2110

CALCULUS-II

SEMESTER-I

TIME 3 HOURS MM: 40

Note: Attempt Two questions each from section A and section B. Section C is compulsory.

Section A (6X2=12)

Q1 i) Discuss the continuity and differentiability of the function $f(x) = \begin{Bmatrix} x\sin\frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{Bmatrix}$

ii) Using $\epsilon - \delta$ definition, prove the limit statement: $\lim_{x \to 1} \frac{x^2-1}{x-1} = 2$

Q2 i)Find the asymptotes , critical points and point of inflection of the function $f(x) = \frac{2x^2+x-1}{x^2-1}$

ii) Does the curve $y = x^4 - 2x^2 + 2$ have any horizontal tangent ? If so, where?

Q3 i) Find the values of x for which $y = x^4 - 6x^3 + 12x^2 + 5x + 7$ is concave upwards or concave downwards.

ii) a) Evaluate :$\lim_{t \to 0} \frac{\sin(1-\cos t)}{1-\cos t}$ b) Find $\frac{d^2y}{dx^2}$ $for\ x^{\frac{2}{3}} + y^{\frac{2}{3}} = 1$

Q4. Trace the curve giving all the necessary details about the curve: $y = \frac{x^3+1}{x}$

Section B (6X2=12)

Q5.i) Let $f(x,y) = \begin{Bmatrix} 1 & if\ x\ is\ irrational \\ 0 & if\ x\ is\ rational \end{Bmatrix}$. Show that for any point (x,y), $\lim_{(x,y)\to(x_\circ,y_\circ)} f(x,y)$ does not exist.

ii) If $z = \varphi(y + ax) + \psi(y - ax)$, show that $\frac{\partial^2 z}{\partial x^2} = a^2 \frac{\partial^2 z}{\partial y^2}$.

Q6 Find the extreme values of the function $f(x,y) = (x-y)^4 + (y-1)^4$

Q7.i) Expand y^x upto second degree terms at (1,1).

ii) If $u = e^x \sin y, x = \log t, y = t^2$, find $\frac{du}{dt}$ by partial differentiation.

Q8. If $= \tan^{-1}\left\{\frac{x^3+y^3}{x-y}\right\}$, using Euler's theorem find the values of $x^2\frac{\partial^2 z}{\partial x^2} + 2xy\frac{\partial^2 z}{\partial x \partial y} + y^2\frac{\partial^2 z}{\partial y^2}$

Section C (8X2=16)

Q9i) Find the critical points of $y = x^{\frac{5}{3}} - 5x^{\frac{2}{3}}$

ii) What are different types of discontinuities of a function. Define g(4) in a way that extends $g(x) = \frac{x^2-16}{(x^2-3x-4)}$ to be continuous at x=4.

iii) State Leibnitz theorem.

iv) If $p^2 = a^2cos^2\theta + b^2sin^2\theta$, prove that $p + \frac{d^2p}{d\theta^2} = \frac{a^2b^2}{p^3}$

v) If $(x, y) = \sqrt{1 - 2xy + y^2}$. Evaluate $f_x(1,3) and\ f_y(1,2)$

vi) Define saddle point. Also give an example of a function with saddle point.

vii) State necessary and sufficient condition for a function of more than two variables to have maxima and minima at a point.

viii) Show that $\lim_{(x,y)\to(0,0)} \frac{2x^2y}{x^4+y^2}$ does not exist.