A-S-2110

DIFFERENTIAL EQUATIONS-II
SEMESTER-I

TIME 3 HOURS

MM: 40

Note :The candidates are required to attempt two questions each from Section A & B, Section C will be compulsory.

Section-A

I (a) $If\ M\ dx + N\ dy = 0\ is\ a\ non\ exact\ homogeneous\ differential\ equation\ and\ Mx + Ny \neq 0, then\ prove\ that\ the\ integrating\ factor\ is\ \frac{1}{Mx+Ny}$ (3)

I (b)Solve the following differential equation

$$\cos x \cos y dx - 2 \sin x \sin y dy = 0$$ (3)

II (a) Solve by method of variation of parameters the differential equation

$$\frac{d^2y}{dx^2} + 9y = \sec 3x$$ (3)

II (b) Solve the following differential equation

$$\frac{d^4y}{dx^4} + y = \cos x$$ (3)

III (a) Solve the following differential equation

$$\frac{dy}{dx} = \frac{x+y+4}{x-y-6}$$ (3)

III (b) Solve the following differential equation

$$\sqrt{1 + x^2 + y^2 + x^2y^2} + xy\frac{dy}{dx} = 0$$ (3)

IV (a) Solve the differential equation $\sqrt{1 - y^2}dx = [\sin^{-1}(y) - x]dy$ (3)

IV (b)If Wronskian of functions $f_1, f_2, f_3, \ldots, f_n$ is non zero over an interval Then $f_1, f_2, f_3, \ldots, f_n$ are linearly independent over that interval (3)

Section-B

V (a) Solve the following differential equation

$$x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 4y = \cos(logx), x > 0$$ (3)

V (b) Solve the System by Using Operator Method

$\frac{dx}{dt} + \frac{dy}{dt} - 2x - 4y = e^t$ and $\frac{dx}{dt} + \frac{dy}{dt} - y = e^{4t}$ (3)

VI Solve in series the legendre equation $(1 - x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$ (6)

VII If α *and* β *are roots of* $J_n(ax) = 0$, *then prove that*

$\int_0^r xJ_n(\alpha x)J_n(\beta x)dx = 0$ Where $\alpha \neq \beta$ *and* r *is any fixed real number* (6)

VIII (a) Show that $\int_{-1}^{1}(1 - x^2)\, P_m{}' P_n{}' dx = 0$,

where m *and* n *are distinct positive integers.* (3)

VIII (b) Prove that $P_n{}'(-1) = (-1)^{n-1}\frac{n(n+1)}{2}$ (3)

Section-C

IX (a) Show by Wronskian that the following functions are linearly independent

e^x, e^{2x} and e^{3x} for all real x

IX (b) Define order and degree of a differential equation

IX (c)) Prove that $\frac{1}{D-a}V = e^{ax}\int Ve^{-ax}\,dx$, no arbitrary constant being added

IX (d) Solve the differential equation $x\frac{dy}{dx} + y = x\log x$

IX (e) State Rodrigues' Formula

IX (f) Express $J_4(x)$ in terms of $J_0(x)$ and $J_1(x)$

IX (g) Define ordinary point of linear equation of second order

IX (h) Solve the differential equation $\frac{d^3y}{dx^3} = \frac{6y}{x^3}$

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(2x8=16)