AS/2110 CALCULAS-1, Paper-I 5194/NH

Semester-I

Time Allowed:- Three Hours] [Maximum Marks:40

Note:- The Candidate are required to attempt two questions each from Sections A and B carrying 6 marks each and the entire Section C consisting of eight short answer type questions carrying 2 marks each.

Section A

I. (a) If the function $f(x)$ defined below is continuos at $x = 0$. Find the value of k.

$$f(x) = \begin{cases} \frac{1-cos2x}{2x^2}, x < 0 \\ k, \ x = 0 \\ \frac{x}{|x|}, \ x > 0 \end{cases}$$ (3)

(b) If $y = Acosnx + Bsinnx$, Prove that $\frac{d^2y}{dx^2} + n^2y = 0$. (3)

II. If $p^2 = a^2cos^2\theta + b^2sin^2\theta$, show that $p + \frac{d^2p}{d\theta^2} = \frac{a^2b^2}{p^3}$. (6)

III. (a) Find all the asymptotes of the curve $y^3 + x^2y + 2xy^2 - y + 1 = 0$. (3)

(b) Find the intervals in which the curve $y = (x^2 + 4x + 5)e^{-x}$ is concave upwards or downwards. Find the points of inflexion. (3)

IV. Trace the curve $x = acos^3t, y = asin^3t$. (6)

Section B

V. (a) Discuss the continuity of the following function at (0,0)

$$f(x,y) = \begin{cases} xysin\frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$ (4)

(b) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, show that

$$(\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z})^2 = \frac{9}{(x+y+z)^2}$$ (2)

VI. State and Prove Euler's Theorem on Homogenous Function of three variables. (6)

VII. Let $f(x,y) = \frac{xy}{\sqrt{x^2+y^2}}$ when $(x,y) \neq (0,0)$ and $f(0,0) = 0$, Show that f is continuous at (0,0) and both $f_x(0,0)$ and $f_y(0,0)$ exist but f is not differentiable at (0,0). (6)

VIII. (a) Let $f: R^3 \to R, be\ defined\ by\ f(x,y,z) = xyz$, Determine x, y, z for maximum of f subject to condition $xy + 2yz + 2zx = 108$. (4)

(b) Find first order Partial derivatives of $sin^{-1}\frac{x}{y}$. (2)

Section C

IX. (a) If $y = a^{mx}$ then $y_n = m^na^{mx}(loga)^n$

(b) State Leibnitz's theorem.

(c) Show that $\lim_{(x,y)\to(0,0)} \frac{x^4y^4}{(x^2+y^4)^3}$ does not exist.

(d) If z= $xytan\frac{y}{x}$, Prove that $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 2z.$

(e) State Schwarz's Theorem.

(f) If $f(x) = ax^3 + 3bx^2$, Determine a and b so that the graph of f has a point of inflexion at (-1,2).

(g) Find domain of continuity of function $sin^{-1}x + logx.$

(h) State Euler's theorem on Homogenous function of two variables. (8x2=16)

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Please continue on another ordinary white foolscap paper sheet, if necessary.