F-40A/2110 11077/NJ

LINEAR ALGEBRA-302

SEM-III
Syll-Dec-2017

TIME: 3hrs M.M.- 75

Note: The candidates are required to attempt two questions each from Section A & B Section C will be compulsory

Section A

1. Solve the following equation:

$$2x + y + z = 10\,;\quad 3x + 2y + 3z = 18;\quad x + 4y + 9z = 16$$

2. Show that

$$\begin{vmatrix} 1 & 1 & 1 \\ \alpha & \beta & \gamma \\ \beta\gamma & \gamma\alpha & \alpha\beta \end{vmatrix} = (\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)$$

3. For the linear transformation $T: R^3 \to R^3$ is defined by

$$T(x,y,z) = (x+2y, y-z, x+2z)$$

Verify the Rank (T) + Nullity (T) = 3.

4. Examine whether the following set of vectors in $V_3(R)$ forms a basis or not: $(1,0,-1)$, $(1,2,1)$, $(0,-3,2)$.

Section B

5. Find all the eigen values & eigen vectors of the matrix $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$ over real R.

6. Reduce the quadratic form

$$2x_1x_2 + 2x_1x_3 - 2x_2^2 + 4x_2x_3 - x_3^2$$

to diagonal form.

7. Verify the Cayley Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

8. Apply the Gram-Schmidt orthonormalization to the following sequence of vectors in R^3:

$$\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix},\quad \begin{bmatrix} 8 \\ 1 \\ -6 \end{bmatrix},\quad \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

P.T.O.

Section C

9. Write in brief:

a) Write the symmetric matrix of the quadratic form $x_1^2 - 2x_2^2 + 3x_3^2 - 4x_2x_3 + 6x_3x_1$.

b) Show that the map $T: R^2 \to R^3$ defined by $T(x, y) = (x + 1, y - 3, y)$ is not linear transformation.

c) Let $W = \{(a, b, c) | b + c + a = 0\}$ be a subspace of R^3. Find the dimension of W.

d) State Cayley-Hamilton theorem.

e) State Rank-Nullity theorem.

f) Define vector space.

g) Define Eigen value and Eigen vector.

h) Define rank of a matrix.

i) If $A = \begin{bmatrix} 2 & 1 \\ -3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & -1 \\ 3 & -3 \end{bmatrix}$. Find $A - B$.

j) Find the rank of matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 3 & 1 & 1 \end{bmatrix}$.

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