M-73/2110

MATHEMATICAL ANALYSIS MM 402/AMC -102

SEMESTER- I, (Dec. 2019)

TIME ALLOWED 3 Hrs

MM 70

NOTE: **The candidates are required to attempt two questions each from Section A & B Section C will be compulsory.**

Section A

Q1: State and prove inverse function theorem.

Q2: A linear operator T on a finite dimensional vector space X is one to one if and only if the range of T is all of X.

Q3: Show that the outer measure of an interval is its length.

Q4: If m is countably additive, translation invariant measure defined on σ algebra containing the set P. Then the set [0, 1) is not measurable.

Section B

Q5: State and prove Fatou's Lemma

Q6: State and prove Vitali's lemma.

Q7: If f is of bounded variation on [a, b], then f'(x) exists almost everywhere on [a, b].

Q8: If f is absolutely continuous on [a, b] and f'(x) = 0 almost everywhere, then f is constant.

Section C

Q9: a) Define absolute continuity.

b) Define basis of vector space X.

c) Define linear independence of vectors in vector space X.

d) Define outer measure of set E.

e) State Egoroff's theorem.

f) If m*E = 0, then E is measurable.

g) If f is bounded measurable function defined on E, then $\left|\int_a^b f\right| \leq \int_a^b |f|$.

h) Define simple function and characteristic function.

i) State Jensen's inequality.

j) Define convex functions.