M-73/2110

ALGEBRA-I- MM 401/ AMC 101

SEMESTER-I (SYLLABUS DECEMBER-2019)

TIME ALLOWED 3 Hrs M.M 70

NOTE: The candidates are required to attempt two questions each from Section A & B carrying 10 marks each. Section C will be compulsory carrying 30 marks.

Section A

1. State and prove Jordan-Holder theorem.
2. Prove that every nilpotent group is solvable, but the converse is not true.
3. Prove that a group of order p^n has a composition series of length n.
4. Show that the alternating group A_n is simple if $n \geq 5$. Deduce that S_n is not solvable for $n \geq 5$.

Section B

5. Prove that if each non-identity element of a finite group G is of order 2 then $o(G) = 2^n, n \geq 1$ and $G \cong C_1 \times C_2 \times \ldots \times C_n$ where C_i are cyclic and $o(C_i) = 2$.
6. State and Prove first Sylow theorem.
7. Let $o(G) = pq$ where p, q are distinct prime, $p < q$ and p does not divide $(q-1)$ then prove that G is cyclic. Using this, check whether groups of order 22 and 35 are cyclic or not.
8. If R is a non-zero ring with unity 1 and A any ideal of R such that $A \neq R$. Then, prove that there exists a maximal ideal M of R such that $A \subseteq M$.

Section C

9. Write in short:

a) Show that a group of order p^n (p is a prime) is nilpotent.

b) Define a composition series of a group G.

c) Find all the non-isomorphic abelian group of order 36.

d) Find a composition series for the group $G = \dfrac{Z}{<6>}$.

e) Prove that the derived group of S_n is A_n.

f) Write down all the elements of S_3.

g) Prove that no group of order 15 is simple.

h) Prove that any group of order $2p$ must have a normal subgroup of order p, where p is prime.

i) Define maximal and prime ideals.

j) Prove that a group of prime order is cyclic.