Roll No. .... **Total Pages : 3**

10701/NH

C-2110

ALGEBRA-I

Paper-I

Semester-V

Syllabus-(Dec-19)

Time allowed : 3 Hours] [Maximum Marks : 40

Note : The candidates are required to attempt two questions each from Section A and B carries 6 marks each. Section C will be compulsory carries 16 marks.

SECTION-A

1. Let G be abelian group, then prove that $(ab)^n = a^n b^n$, for all integers n and for all $a,b \quad G$.

2. Prove that Intersection of two subgroups is a subgroup. Is union of two subgroups also a subgroup. Justify your answer with a suitable example.

3. (i) Prove that Quotient group of a cyclic group is cyclic.

   (i) Prove that A_4 has no subgroup of order six.

4. State and prove third fundamental theorem on Homomorphism.

SECTION-B

5. What is the characteristics of a ring? Prove that characteristic of a field is either 0 or is a prime number. Give an example of a field of characteristic 7.

6. Let I and J be two ideals of a ring R so that I J, then $= \dfrac{(R/I)}{(J/I)} = \dfrac{R}{J}$

7. What is a PID? Prove that every irreducible element in a PID is prime.

8. For any two ideals I and J of the ring R, I J is an ideal of R iff either $I \quad J$ or $J \quad I$.

SECTION-C

9. (i) Prove that every cyclic group is abelian.

(ii) Prove that intersection of two normal subgroups of G is a normal subgroup of G.

(iii) State Lagrange's theorem. Comment whether converse of Lagrange's theorem is true for cyclic groups or not.

(iv) Let G be a group and H be a subgroup of G, show that any two cosets of H in G are either disjoint or identical.

(v) Prove that intersection of two left ideal of a ring is left ideal.

(vi) Prove that centre of a ring R is a subring of R.

(vii) Prove that any field is an integral domain.

(viii) Define principal ideal ring and UFD.