M-52/2110

Discrete Mathematics

10561/N

BCA-213

M.M. 75 **Time: 3 hrs**

Note: Candidates are required to attempt **Five** questions in all by selecting at least **Two** questions each from the section A and B. Section C is compulsory.

Section-A

$2 \times 15 = 30$ marks

Q1: (a) There are exactly three types of students in a school: the hockey players, the football players, and the athletes. Each student is classified into at least one of these categories. And the total number of students in the school is 1000. Suppose that the following is given: The total number of students who are the hockey players is 310. The total number of students who are the football players is 650. The total number of students who are athletes is 440. The total number of students who are both the hockey players and the football players is 170. The total number of students who are both the hockey players and athletes is 150. The total number of students who are both the football players and athletes is 180. What is the total number of students who fit into all 3 categories and the number of students who are only athletes?

(b) State and prove De-Morgan's law.

Q2: (a) Prove that conjunction distributes over disjunction.

(b) Prove by the principle of induction: $1 + 3 + 5 + \ldots + (2n - 1) = n^2$.

Q3: (a) Test the validity: If i work hard then i will be successful. If i am successful then i will be happy. Therefore hard work leads to happiness.

(b) Find the domain and range of the relation given by R:

$$\left\{(x, y): y = x + \frac{6}{x}, where\ x, y \in N\ and\ x < 6\right\}.$$

Q4: (a) Let R be the relation on the set $\{0,1,2,3\}$ containing the ordered pairs (0,1), (1,1), (1,2), (2,0), (2,2), and (3,0). What is the reflexive closure, symmetric closure and transitive closure of R?

(b) Partition $A = \{0,1,2,3,4,5\}$ with the minsets generated by $B_1 = \{0,2,4\}$ and $B_2 = \{1,5\}$ and also find out how many different subsets of A can you generate from B_1 and B_2?

Section-B

2 X 15 = 30 marks

Q5: (a) Explain in detail the representation of the undirected graphs and also give one example.

(b) State and prove Euler theorem.

Q6: (a) A function $f: A \to B$ has an inverse iff f is one-one and onto.

(b) If $f: A \to B$ and $g: B \to C$ are two bijections, then show that $(gof)^{-1} = f^{-1}og^{-1}$.

Q7: (a) Prove that in a complete graph the number of edges is $n(n-1)/2$.

(b) Draw a graph (i) in which no edge is a cut edge. (ii) in which every is a cut edge.(iii) in which there is only one cut vertex.

Q8: (a) A finite connected graph is Eulerian iff each vertex has even degree.

(b) Let G be a finite graph with n>1 vertices. Then following are equivalent: (i) G is tree. (ii) G is cycle free and has n-1edges. (iii) G is connected and has n-1 edges.

Section-C

15 Marks

Q9: (i) Define Cartesian product of sets. Is Cartesian product commutative? Justify. (3)

(ii) Define union and intersection of two sets and give one example of each. (3)

(iii) Define an equivalence relation and give an example. (1)

(iv) Define Big-O Notation. (1)

(v) Define Floor function. (1)

(vi) Explain the Shortest path problem. (3)

(vii) Explain the Prim's Algorithm for minimum spanning tree. (3)

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