M-39/2110

NUMERICAL ANALYSIS-1, PAPER-MM 605/AMC-312

SEMESTER-III (SYLLABUS DECEMBER-2019)

TIME ALLOWED 3 Hrs

M.M 70

NOTE: **The candidates are required to attempt two questions each from Section A & B Section C will be compulsory.**

Section A

1. Given the following value of $f(x)$ and $f'(x)$

x	$f(x)$	$f'(x)$
-1	1	-5
0	1	1
1	3	7

Estimate the value of $f(-0.5)$ and $f(0.5)$ using the Hermite interpolation.

2. $S_3(x)$ is the piecewise cubic Hermite interpolating approximant of $f(x) = \sin x \cos x$ in the abscissas 0, 1, 1.5, 2, 3. Estimate the error $\max|f(x) - S_3(x)|$ over $0 \le x \le 3$.

3. (a) Prove that for $m \geq 2$, and the knots $y_i \leq y_{i+m}$,

$$Q_i^m(x) = \frac{(x - y_i)Q_i^{m+1}(x) + (y_{i+m} - x)Q_{i+1}^{m-1}(x)}{(y_{i+m} - y_i)}$$

for all $x \in R$, where Q_i^m represents the mth order B-spline associated with the knots $y_i, \dots, y_{i+m}$.

(b) Let $y_i < y_{i+m}$ and suppose D_+ is the right derivative operator. Then prove that

$$D_+Q_i^m(x) = \frac{(m-1)\left[Q_i^{m-1}(x) - Q_i^{m+1}(x)\right]}{(y_{i+m} - y_i)}$$

4. Let $y_1 \leq \cdots \leq y_{n+m}$ be such that $y_i < y_{i+m}$, $i = 1,2,\dots,n$. Let $\{N_i^m\}, i = 1,2,\dots n$ be the associated normalized B-splines. Then prove that there is a dual set of linear functional $\lambda_1, \lambda_2, \dots, \lambda_n$ with

$$|\lambda_j f| \leq (2m+1)9^{m-1}h_j^{-1/9} \|f\|_{L_p[I_j]}, \quad 1 \leq p < \infty$$

where $I_j = (y_j, y_{j+m})$ and $h_j = y_{j+m} - y_j$, $j = 1,2,\dots,n$

Section B

5. Let $\Delta = \{a = x_0 < x_1 < x_2 < \cdots < x_k < x_{k+1} = b\}$ be a partition of the interval $[a, b]$. Then prove that there exists an associated partition $\Delta^* = \{a = x_0^* < x_1^* < \cdots < x_l^* < x_{l+1}^* = b\}$ with $\Delta^* \subseteq \Delta$ such that

$$\frac{\bar{\Delta}}{2} \leq \underline{\Delta}^* \leq \bar{\Delta}^* \leq \frac{3\bar{\Delta}}{2}$$

where $\bar{\Delta}$ is the mesh size of the partition Δ associated with the spline space.

6. Let $\Delta = \{a = x_0 < x_1 < x_2 < \cdots < x_k < x_{k+1} = b\}$ be a partition of the interval $[a, b]$, and let $S_1(\Delta)$ be the corresponding space of piecewise constant functions. Then prove that for any Δ, the following bounds on the approximation order holds.

P.T.O.

(i) $d[f, S_1(\Delta)]_\infty \le \frac{1}{2} w(f; \bar{\Delta})_\infty$, for all $f \in B[a, b]$;

(ii) $d[f, S_1(\Delta)]_\infty \le \frac{1}{2} \bar{\Delta} \, ||Df||_\infty$, for all $f \in C^1[a, b]$

where w describing the smoothness of the function f.

7. Prove that the periodic B-splines form a basis for the space of periodic polynomial splines of order m with knots at $x_1, x_2, \ldots, x_k$ of multiplicity $m_1, m_2, \ldots, m_k$.

8. Show that the functions $\rho_{ij}(x) = \{g_{m-j+1}(x; x_i)\}$ with $j = 1,2, \ldots, m_i; i = 0, 1, \ldots, k$ with $m_0 = m$ is a basis for the space of Tchebycheddian spline functions with knots $x_1, x_2, \ldots, x_k$ of multiplicities $m_1, m_2, \ldots, m_k$.

Section C

9. Write in short.

a) Write the general equation for piecewise cubic Hermite interpolation polynomial.
b) Find a dual basis for $S = span(N_1^1, \ldots, N_n^1)$.
c) Define B-spline
d) Differentiate between piecewise and spline interpolation.
e) What is the difference between perfect B-spline and B-spline.
f) Write the sign properties of green's function.
g) Write short note on dual basis of splines.
h) Define g-splines.
i) Differentiate between periodic splines and natural splines.
j) Define the N-widths of spline.

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