M-39/2110

DIFFERENTIAL EQUATIONS-II, MM 603/ AMC 310

SEMESTER-III, (SYLLABUS DECEMBER-2019)

TIME ALLOWED 3 Hrs

M.M 70

NOTE: **The candidates are required to attempt two questions each from Section A & B carrying 10 marks each. Section C will be compulsory carrying 30 marks.**

Section A

1. State and prove Caratheodory theorem.
2. Let f be analytic in a domain D of the (z, w) space and suppose ψ is a solution of $\frac{dw}{dz} = f(z, w)$ on H, where H is a closed convex domain of the z plane. Then prove that for a $\delta > 0$ such that for any $(\zeta, w) \in U$, where $U: \zeta \in H \ |w - \psi(\zeta)| < \delta$, there exists a unique solution $\psi = \psi(z, \zeta, w)$ of $\frac{dw}{dz} = f(z, w)$ on H with $\psi(\zeta, \zeta, w) = w$.
3. Let D be a domain of (t, x) space, I_μ the domain of $|\mu - \mu_0| < c,\ c > 0$ and D_μ the set of all (t, x, μ) satisfying $(t, x) \in D, \mu \in I_\mu$. Suppose f is a continuous function on D_μ bounded by constant M there. For $\mu = \mu_0$ let $x' = f(t, x, \mu),\ x(\tau) = \xi$, have a unique solution ψ_0 on [a, b], where $\tau \in [a, b]$. Then prove that there exists a $\delta > 0$ such that, for any fixed μ, satisfying $|\mu - \mu_0| < \delta$, every solution I_μ of $x' = f(t, x, \mu),\ x(\tau) = \xi$ exist over [a, b] and as $\mu \to \mu_0$, $\psi_\mu \to \psi_0$ uniformiy over [a, b].
4. Let $f \in C\ (n = 1)$ on the rectangular $0 \le t \le a, |x| \le b$, where $a, b > 0$ and assume $f(t, x_1) \le f(t, x_2)$ if $x_1 \le x_2$ and $f(t, 0) \ge 0$ for $0 \le t \le a$. Prove that the successive approximations converge to a solution of $x' = f(t, x),\ x(0) = 0$, on $0 \le t \le \alpha = min\left(a, \frac{b}{M}\right)$ where $M =$ max $|f|$ on the rectangle.

Section B

5. Find the distribution which gives rise to the potential
$$\psi = \begin{cases} \frac{1}{3}\pi \log R\ ; & R > 1 \\ \pi \log R + \frac{\pi}{9}(5 - 9R^2 + 4R^3)\ ; & R < 1 \end{cases} \quad \text{where } R^2 = x^2 + y^2.$$
6. Check whether the surfaces $(x^2 + y^2)^2 - 2a^2(x^2 - y^2) + a^4 = c$, where c is a parameter form a family of equipotential surfaces or not? If yes, find the general form of the corresponding potential function.
7. Prove that the solution $\psi(r, \theta, \phi)$ of the exterior Dirichlet problem for the unit sphere $\nabla^2 v = 0, r > 1, \psi = f(\theta, \phi)$ on $r = 1$ is given in terms of the solution $v(r, \theta, \phi)$ of the interior Dirichlet problem $\nabla^2 v = 0, r < 1, v = f(\theta, \phi)$ on $r = 1$ by the formula $\psi(r, \theta, \phi) = \frac{1}{r} v\left(\frac{1}{r}, \theta, \phi\right)$.
8. A uniform circular wire of radius 'a' charged with electricity of line density 'e' surrounds concentric spherical conductor of radius 'c'. Determine the electrical charge density at any point on the conductor.

Section C

9. Write in brief:
   a) Define equipotential surface.
   b) State Green's theorem for Laplace function.
   c) State Copson's theorem.
   d) How the interior Dirichlet boundary value problem for Laplace equation differ from their exterior Dirichlet boundary value problem.
   e) State only existence theorem of solutions of first order differential equation for complex systems.
   f) State maximum and minimum solutions of differential equation $x' = f(t, x)$ with $x(\tau) = \xi$.
   g) State Kelvin's Inversion theorem.
   h) Write a short note on axial symmetry.
   i) Write a short note on the successive approximation for the differential equation.
   j) Show that the continuity of a function f is not sufficient for the convergence of the successive approximations.