

M-39/2110

10492/N

FIELD THEORY (MM602/AMC 309)

(Semester-3)

(Syll-Dec-2019)

Time: 2 hours

M.M. 70

The students are required to attempt four questions. All questions will carry equal marks.

Q.1 (a) Prove that finite extension is algebraic. Is the converse true? Justify your answer.(9)

(b) Let $F \subset K \subset E$ be fields such that K is algebraic extension of F and E is algebraic extension of K . Prove that E is algebraic extension of F . (8.5)

Q.2 (a) Find the splitting field of x^4-2 over \mathbb{Q} . Also find the degree of splitting field over \mathbb{Q} .(9)

(b) Show that a finite extension E of field F is a normal extension if and only if E is a splitting field of a polynomial $f(x) \in F[x]$. (8.5)

Q3 (a) Prove that in any finite field any element can be written as the sum of two squares.(9)

(b) Let K be a field of characteristic p , then K is perfect if and only if $K^p = K$. (8.5)

Q.4(a) If E is a finite separable extension of F then prove that E is a simple extension of F .(9)

(b) If $w = e^{\frac{2\pi i}{n}}$ and $u = \cos \frac{2\pi}{n}$. Prove that $[Q(w):Q(u)]=2$. (8.5)

Q.5(a) Let E be a finite normal and separable extension of F . Prove that F is fixed field of $G(E/F)$. (9)

(b) Prove that the Galois group of $x^3 - 2$ over \mathbb{Q} is the group of symmetries of the triangle.(8.5)

Q.6 State and Prove Fundamental Theorem of Algebra . (17.5)

Q.7 Suppose that F contains a primitive m th root of unity. Prove that E is cyclic extension of F of degree m if and only if E is the splitting field of an irreducible polynomial x^m-b over F . (17.5)

Q. 8 (a) Let G be a transitive group of permutations of S_p , p prime, containing a transposition (ab) , then prove that $G = S_p$. (9)

(b) Show that the polynomial $x^5 - 9x + 3$ is not solvable by radicals. (8.5)

Q.9(a) Express $x_1^3+x_2^3+x_3^3$ as rational function of elementary symmetric function.

(b) Prove that $Q(\sqrt{2}, \sqrt{3})=Q(\sqrt{2} + \sqrt{3})$

(c) Construct field with 16 elements.

(d) Show that any extension of degree 2 is normal extension.

(e) Find 8th cyclotomic polynomial.

3.5×5=17.5

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