Roll No. .... **Total Pages : 4**

10395/NH

CS/2110

ALGEBRA

Paper-I

Semester-V

Syllabus-(Dec-19)

Time allowed : 3 Hours] [Maximum Marks : 40

Note : The candidates are required to attempt two questions each from Section A and B. Entire Section C is compulsory.

SECTION-A

1. State and Prove Lagrange's theorem. Also prove that converse of Lagrange's theorem is not true. 6

2 State and prove fundamental theorem of isomorphism. 6

3. (i) If H is a subgroup of the group G and a G then Ha = H iff a H. 3

(ii) If G is a group such that G/Z(G) is cyclic, where Z(G) is the center of G, then G is abelian. 3

4. (i) A subgroup of a cyclic group is cyclic. 3

(ii) Intersection of two normal subgroups of a group G is again a normal subgroup of G. 3

SECTION-B

5. Prove that intersection of two Ideals is again an ideal but union may not be an ideal. Also discuss the case when the union of two ideals will be again an ideal. 6

6. (i) Prove that center of a ring is a Subring. 3

(ii) If f is a homomorphism of ring R into R' then prove that Kerf is an ideal of ring R. 3

7. Show that the ring Z[i] of Gaussian integers is an Euclidean domain. 6

8. (i) In a ring R with unity, $(xy)^2 = x^2y^2$ for all x, y R then show that R is commutative. 3

(ii) Prove that 5Z is an ideal of ring Z, integers and hence find all the elements of quotient ring Z/5Z. Also write their composition table. 3

SECTION-C

9. (i) Prove that for all a G if $a^2 = e$, then the group G is abelian.

(ii) Prove that 3^{rd} roots of unity form an abelian group.

(iii) Define field of quotients of an integral domain.

(iv) Prove that subgroup of an abelian group is always normal.

(v) Prove that in a group identity element is unique.

(vi) Let R be a ring with unit element 1. If I is an ideal of R and 1 I then I = R.

(vii) Define division ring.

(viii) Let R be a ring. Define f: R R by f(a) = a, a R, show that f is a ring homomorphism and kerlf = {0}. 8x2 = 16