BS/2110

ANALYSIS-1, Paper-II

Semester-III

10377/NH

Time Allowed:- Three Hours] [Maximum Marks:40

Note:- The Candidate are required to attempt two questions each from Sections A and B carrying 6 marks each and the entire Section C consisting of eight short answer type questions carrying 2 marks each.

Section A

I. If $\{a_n\}$ is a sequence such that $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = l$ and $|l| < 1$ then $\lim_{n\to\infty} a_n = 0$. (6)

II. (a) Prove that the sequence $\{a_n\}$ where $\{a_n\} = \frac{1}{n+1} + \frac{1}{n+2} + \frac{1}{n+3} + \cdots + \frac{1}{2n}$ is convergent. (4)

(b) Show that $\lim_{n\to\infty} \frac{1}{n}(1 + \frac{1}{2^2} + \cdots + \frac{1}{n^2}) = 0$. (2)

III. (a) Test for convergence or divergence of the series

$\frac{2}{1^2}x + \frac{3^2}{2^3}x^2 + \frac{4^3}{3^4}x^3 + \cdots, x > 0$. (3)

(b) Discuss the convergence of the following series

$1 + \frac{x}{2^2} + \frac{x^2}{3^2} + \frac{x^3}{4^2} + \cdots, x > 0$. (3)

IV. (a) If a series $\sum_{n=1}^{\infty} a_n$ is absolutely convergent, then it is convergent also. (3)

(b) Examine the convergence or divergence of the series

$\frac{1}{2} + \frac{1.3}{2.4} + \frac{1.3.5}{2.4.6.} + \cdots$ (3)

Section B

V. (a) Complete the lower sum $L(P, f)$ of the function $f(x) = x^2$ defined on [0,1] for the partition $P = \left\{0, \frac{1}{n}, \frac{2}{n}, \ldots \frac{n}{n}\right\}$ and find $\lim_{n\to\infty} L(P, f)$. (4)

(b) Evaluate $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (Sin|x| - Cos|x|)dx$. (2)

VI. If f is bounded and integerable in [a,b], then $|f|$ is also bounded & integerable in [a,b]. Moreover $\left|\int_a^b f\, dx\right| \le \int_a^b |f|\, dx$. (6)

VII. (a) Using Mean Value Theorem Estimate the value of $\int_0^1 \frac{Sinx}{1+x^2} dx$. (3)

(b) Show that the function $f: [0,1] \to R$ defined by $f(x) = \begin{cases} x^2 cos\frac{1}{x}, & x \ne 0 \\ 0, & x = 0 \end{cases}$ is of bounded variation on [0,1]. (3)

VIII. A function f of bounded variation on [a,b] is continuous iff V is continuous, where V is a variation function. (6)

Section C

IX. (a) Show that $\left\{(1 + \frac{1}{n})^n\right\}$ is bounded.

(b) Give an example of divergent sequence $\{a_n\}$ & $\{b_n\}$ such that sequence $\{a_n + b_n\}$ is not divergent.

(c) Show that $\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = 1$.

(d) Discuss the convergence or divergence of the following series $\sum_{n=2}^{\infty} \frac{1}{logn}$.

(e) State Cauchy's root test.

(f) Define absolute convergence of a series. Give example

(g) Find the lower and upper sum for the function $f(x) = \begin{cases} 0, & when\ x\ is\ rational \\ 1, & when\ x\ is\ irrational \end{cases}$ $on\ [-1,1]$ by dividing it into n equal sub intervals.

(h) Prove that a function of bounded variation is bounded. (2x8=16)