F-57/2110 10320/NJ

CSM 352: Abstract Algebra
Part-III Semester 5
(Syll-Dec-2019)

Time Allowed: 2 Hours **Maximum Marks: 45**

Note: - Attempt any ***four*** questions.Each question carries equal marks.

Q1.a) Show that $G = \left\{\begin{pmatrix} a & a \\ b & b \end{pmatrix}\right\}: a, b\ reals, a + b \neq 0\}$is a semigroup under matrix multiplication. Also Check whether G is a group or not? Justify your answer.

b) Check whether G = {2, 4, 6, 8} under multiplication modulo 10 is a Group ?

Q2.a) Write all symmetries of rectangle. Show that these symmetries form Klein 4-group.

b) If G is non-Abelian group then show that Aut(G) is not cyclic group.

Q3.a)Find the Kernel of the homomorphism $f: Z \longrightarrow Z_n$ given by $f(x) = \bar{x}$.

b) Show that $Z \times Z$is not cyclic group.

Q4.a) Give an example of a group G having subgroups H and K such that H is normal in K and K is normal in G but K is not normal in G.

b) Prove that the set of even permutations in $S_n$forms a subgroup of S_n.

Q5.a) Verify Cayley's Theorem for group G = { 1, -1, i, - i}.

b) If $f: G \longrightarrow G$ s.t $f(x) = x^n$is an automorphisms of G.Then show $a^{n-1} \in Z(G)\ \forall\ a \in G$.

Q6. Show that a set of endomorphisms of an abelian group forms a ring with unity.

Q7.Show that every Euclidean Domain is a Unique Factorization Domain.

Q8.Let R be a PID which is not a field. Show that an ideal A is maximal if and only if A is generated by irreducible element.

Q9. Let R be a commutative ring with unity. Show that

i. If $a \in R$ is a unit then $a$is not nilpotent.

ii. If $a \in R$ is nilpotent then 1+ $a$is a unit

iii. The sum of nilpotent and unit elements is a unit.