F-56/2110 **10315/NJ**

CSM-232/ DIFFRENTIAL EQUATIONS

(SEM 3rd)

(Syll-Dec-2019)

Note : Do any 4 Questions

1. Solve the differntial equation $(1-x^2)\, d^2y/d^2x + 2y = 0$ in power series about '0' .
2. Verify the Legendre polynomial $P_3(x) = (5/2)\, x^2 - (3/2)\, x$ satisfies the Legendre's equation when n= 3.
3. (a) Solve the partial differential equation $(x-a)p+(x-b)q = (z-c)$.
   (b) Show that $y = x^n J_n(x)$ is the solution of $x\, d^2y/d^2x + (1-2n)dy/dx + xy = 0$.
4. Solve the partial diiferential equation $p^2 + q^2 = \alpha^2$
5. If F(s) is the Laplace Transformation of f(t) for $t \geq 0$ and α is any number (real or complex).Prove that the function $g(t) = \begin{cases} f(t-\alpha), & t > \alpha \\ 0, & t < \alpha \end{cases}$ has Laplace transform $e^{-\alpha}$ F(s) .
6. (a) Prove that $\int_0^\infty \sin x^2\, dx = \sqrt{(\pi/8)}$ for x> 0 .
   (b) If L(f(t)) = F(s) for $t \geq 0$ .Prove that for any positive constant α, $L(f(\alpha t)) = \frac{1}{\alpha} F(\frac{s}{\alpha})$.
7. Solve $(D^2+1)x = a\sinh t$, where x(0)=0 and $x'(0) = 0$
8. Solve $2r-s-3t = 5e^{x-y}$.
9. (a) Express $x^4+4x^3-5x^2+x-3$ in the terms of Legendre's polynomial.

   (b) Prove that $z = \frac{y}{x}$ is the solution of the partial differential equation px+qy = 0

   (c) Evalute $L(t\cos \alpha t)$,$t \geq 0$

   (d) Show that $\int_0^\infty e^{-3t}\, t \cos t = \frac{2}{25}$

   (e) Find $L^{-1}(\frac{5s-8}{4s2+36})$

   (f) Solve $\frac{dy}{dt}+y = 0$, $\frac{dy}{dt}+x = 2\cos t$ Given that x(0) = 1 ,y(0) = 0

   (g) Solve the PDE yzp + zxq = x for the general solution .