F-56/2110 10314/NJ

CSM-231 ADVANCE CALCULUS

(SEM 3rd)

(Syll-dec-2019)

Note : Do any 4 Questions

1. State and prove Cauchy's Second theorem on limits.
2. (a) Show that series $\sum_{k=1}^{n} 1/(n+k)^2$ converges to zero .

   (b) Prove that the Sequence $\{\frac{3n+1}{4n+5}\}$ is bounded

3. Discuss the convergence or divergence of series Σa_n where $a_n = e^{\sqrt{n}}.r^n$.

4. If Σa_n is the positive terms convergent series , then show that Σa_n^2 is convergent . Is

   the converse true? Justify your answer.

5. State and prove Dirichlet's test on series.

6. Discuss the convergence or divergence of the series $\Sigma 1/n^p$ where p >0.

7. If f(x) = x^3-2x+5 .Find the value of f(2.001)with the help of Taylor's Theorem .Find

   the an Approximate value of f(x) when x changes from 2 to 2.001

8. Verify the Rolle's Theorem for the function f(x) = cos $2(x-\frac{\pi}{4})$ in the interval $[0, \frac{\pi}{4}]$.

9. (a) Show that the function f(x) = x^2+3x+2 is uniform continous in the closed interval

   [1,2]

   (b) If a sequence is convergent ,Then prove that it converges to a unique limit.

   (c) Prove that $\log_{n\to\infty}\{An\} = 27$ where An $=\frac{3!}{(n!)3}$.

(d) Verify Lagrange's Mean Value theorem for the funvtion f(x)= $2x^2-10x+29$ in the

interval [2,7]

(e) Show that the equation $x^{41}+x+1 = 0$ has a real root.

(f) Prove that a convergent sequence is always a Cauchy sequence.

(g) Prove that the function $f(x) = \begin{cases} \frac{x}{|x|-2x}, & x \neq 0 \\ k, & x = 0 \end{cases}$ remains discontinuous at

x=0

regardless of choice of k

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