Set-B F-43/2110

10059/NJ

BMH 501: Algebra II

Time Allowed: 3 Hours **Maximum Marks: 75**

Note: - Attempt any two questions each from section-A and section-B. Section-C is compulsory.

Section-A

2×11.25=22.5

Q.1.a)If H and K are subgroups of a group G, prove that if $H \subseteq K$ then $H' \subseteq K'$

b) Any finite p-group is solvable.

Q.2. State and prove Jordan-holder Theorem for finite groups.

Q.3. a) Let R be a commutative ring. Prove that the set of all nilpotent elements of R is an ideal of R.

b) Prove that in Z, an ideal (n) is maximal if and only if n is prime.

Q.4.a)Show that a commutative ring R with unity is a field if and only if R has no proper ideal.

b)Prove that every maximal ideal in a commutative ring with unity is a prime ideal.

Section-B

2× 11.25 = 22.5

Q.5. State and prove fundamental theorem of Ring Isomorphism.

Q.6. Show that for any vector space V, $Hom(V, V)$ is a ring with unity.

Q.7. Prove that every PID is a UFD.

Q.8. (a) Show that every irreducible element of ED is prime element.

(b) Show that $Z[\sqrt{-7}]$ is not UFD.

Section-C

3× 10 = 30

Q.9.

a) If in a group G, $a^5 = \{e\}$ and $aba^{-1} = b^2$ for all $a, b \in G$, prove that if $b \neq e$ then $O(b) = 31$

b) Prove that every group of order p^2q, where p, q are prime is solvable.

c) Prove that for any prime ideal P of ring R, $\frac{R}{P}$ is an integral domain.

d) The union of two ideals may or may not be an ideal. Justify this statement

e) Give an example of a non commutative ring R and an ideal I of R such that quotient R/I is a field.

f) State Correspondence theorem.

g) Show that $Hom(Z^+, Z^+) \cong Z$ where Z is ring of integers.

h) Explain Field of quotients and embedding of a ring.

i) If R is a PID then show that every pair of element of R has an HCF and an LCM.

j) If a, b are non-zero elements of an Euclidean Domain, prove that if a and b are associates then $\delta(a) = \delta(b)$