F-42/2110

LINEAR INTEGRAL EQUATIONS-505

SEMESTER-V

SYLLABUS-DECEMBER- 2019

TIME ALLOWED 3 Hrs M.M 70

NOTE: The candidates are required to attempt two questions each from Section A & B Section C will be compulsory

Section A

1. Reduce the differential equation $y''(x) - 3y'(x) + 2y(x) = 4\sin x$ with the conditions $y(0) = 1, y'(0) = -2$ into an integral equation.

2. (a) Show that the function $\phi(x) = (1+x^2)^{-3/2}$ is a solution of the Volterra integral equation $\phi(x) = \frac{1}{1+x^2} - \int_0^x \frac{\xi}{1+x^2}\phi(\xi)\,d\xi$.

   (b) Find the first two iterated kernels of the kernel $K(x,\xi) = (x-\xi)^2$; $a = -1, b = 1$.

3. Solve the non-homogenous Fredholm's integral equation of the second kind, by the method of successive approximations to the third order. $\phi(x) = 2x + \lambda\int_0^1 (x+\xi)\phi(\xi)d\xi$, $\phi_0(x) = 1$.

4. With the aid of the resolvant kernel, find the solution of the integral equation $\phi(x) = e^{x^2} + \int_0^x e^{x^2-\xi^2}\phi(\xi)\,d\xi$.

2x10=20

Section B

5. Determine $D(\lambda)$ and $D(x,\xi;\lambda)$ for the kernel $K(x,\xi) = x\xi$ with specified limits of a and b as $a = 0, b = 10$.

6. State and Prove Hadamard's theorem.

7. Solve the following integral equation: $\phi(x) = e^x + \lambda\int_0^1 2e^x e^\xi \phi(\xi)d\xi$.

8. Prove that the series $D(\lambda)$ converges absolutely and permanently in λ.

2x10=20

Section C

9. Write in short:

   a) Reduce the initial value problem $y' - 3x^2y = 0,\ y(0) = 1$ to the Volterra integral equation.
   b) Find the resolvent kernel associated with the following kernel $K(x,\xi) = |x-\xi|$ in the interval $(0,1)$.
   c) State Dirichlet's and Neumann's problems.
   d) Write Fredholm's and Volterra linear integral equation.
   e) Show that the integral equation $f(x) = \lambda\int_0^\pi(\sin x \sin 2\xi)f(\xi)d\xi$ has no eigenvalues.
   f) Find the first two iterated kernels of the kernel $K(x,\xi) = (x-\xi)^2$; $a = -1, b = 1$.
   g) Write the Fredholm's fundamental relations.
   h) State Fredholm's first fundamental relation.
   i) State Schwarz's inequality.
   j) Write the difference between $D(\lambda)$ and $D(x,\xi;\lambda)$.

10x3=30