PC-10056/NJ

F-42/2110

DISCRETE MATHEMATICS GRAPH THEORY – 503

(Semester–V)

(Syllabus Dec.-19)

Time : Three Hours] [Maximum Marks : 70

Note : Attempt *two* questions each from Sections A and B carrying 10 marks each. Section C is compulsory and it will be of 30 marks.

SECTION – A

I. (a) How many integers between 0 and 10,000 have exactly one digit equal to 5.

(b) For $0 \leq r \leq n$, show that ${}^{n}C_r = \dfrac{n!}{r!\,(n-r)!}$.

II. (a) Find the number of 8-per-mutations of set $S = \{3.\alpha, 2.\beta, 4.\gamma\}$ of 9 objects of 3 types.

(b) Prove the following identities :

(i) ${}^{11}C_1 + 2\,{}^{n}C_2 + ... + n\,{}^{n}C_n = n\,2^{n-1}$ where $n \geq 1$.

(ii) $\sum_{k=1}^{n} k^2\,{}^{n}C_k = n(n+1)2^{n-2}$, where $n \geq 1$.

III. (a) State and prove Pascal's formula.

(b) Find the number of integers from 1 to 1000 which are divisible by none of 5, 6 and 8.

IV. (a) Solve the recurrence relation $s_n - 6s_{n-1} + 9s_{n-2} = 0$, where $s_0 = 1$, $s_1 = 9$.

(b) Find the generating function of

$$S(n + 2) = S(n + 1) + S(n)$$

where $S(0) = S(1) = 1$ for $n \geq 0$.

SECTION – B

V. Show that a connected general graph possesses an open Eulerian trail iff it has exactly two vertices of odd degree. Also show that every open Eulerian trail joins those vertices. Lastly give an example of a connected graph that has

(i) Neither an Euler circuit nor a Hamiltonian circuit.

(ii) Both an Hamiltonian circuit and Euler circuit.

VI. (a) If G is a graph whose vertices are integers from 1 to 20, two vertices are joined by an edge iff their distance is an odd integer. Prove that G is bipartite graph. Find its bipartition.

(b) Show that a connected graph of order $n(\geq 1)$ is a tree iff it has exactly $n - 1$ edges.

VII. Apply DF-algorithm to determine a DFs-tree for the following :

(i) The graph of the regular dodecahedron (any root).

(ii) Graph Buster (any root).

(iii) A graph of order n whose edges are arranged in a cycle (any root).

(iv) A complete graph k_n (any root).

In each case, determine the depth-first numbers.

VIII. (a) Show that in a general diagraph D (V, A) the sum of in degrees of all the vertices is equal to sum of the out degrees and each is equal to the number of arcs.

(b) If G, is a connected planar graph of order n with e edges. If the plane is divided into r regions by graph G then show that $r = e - n + 2$.

SECTION – C

(Compulsory question)

IX. (i) State Pigeonhole principle : Simple form.

(ii) Define Permutation of sets with example.

(iii) Define combinations of multisets with example.

(iv) State Principle of inclusion and exclusion.

(v) Define linear homogeneous recurrence relation.

(vi) Define multigraph and general graph.

(vii) Define a trail, a chain, cycle.

(viii) Define Bipartite multigraph.

(ix) State the Algorithm of a spanning tree.

(x) Define diagraph. Also define indegree and outdegree of a diagraph.