F-42/2110

CALCULUS OF SEVERAL VARIABLES AND IMPROPER INTEGRALS-502

SEMESTER-V

TIME ALLOWED 3 Hrs M.M 70

NOTE: The candidates are required to attempt two questions each from Section A & B Section C will be compulsory

Section A

1. (a) Discuss the continuity of the function $f(x,y)$ defined by

$$f(x,y)=\begin{cases} xy\sin\frac{1}{x} & ; \text{ if } x\neq 0 \\ 0 & ; \text{ if } x=0 \end{cases}$$ at the origin.

(b) If $u_1=\frac{x_2x_3}{x_1}, u_2=\frac{x_3x_1}{x_2}, u_3=\frac{x_1x_2}{x_3}$. Prove that $J(u_1,u_2,u_3)=4$, where J represent the Jacobian.

2. Show that $f_{xy}(0,0)\neq f_{yx}(0,0)$ for the function $f(x,y)$ defined as

$$f(x,y)=\begin{cases} x^2\tan^{-1}\left(\frac{y}{x}\right)-y^2\tan^{-1}\left(\frac{x}{y}\right) & , \text{ if } xy\neq 0 \\ 0 & , \text{ if } xy=0 \end{cases}$$

3. State and Prove inverse function theorem.

4. (a) Show that the function $f(x,y)=|x|+|y|$ is not differentiable at (0,0).

(b) Find the local maxima, local minima and saddle point, if any, of the function $f(x,y)=2xy-5x^2-2y^2+4x-4$

Section B

5. (a) Prove that $\frac{\beta(m+1,n)}{\beta(m,n)}=\frac{m}{m+n}$ if m,n are the positive integer.

(b) Evaluate $\int_0^\infty 2^{-9x^2}\,dx$ using the Gamma function.

6. Show that $\int_0^\infty \frac{\sin x}{x}\,dx$ is convergent, but not absolutely.

7. State and prove Cauchy's criterion for uniform convergence.

8. (a) Test the convergence of the integral $\int_0^\infty \frac{x\tan^{-1}x}{(1+x^4)^{1/3}}\,dx$.

(b) Show that the improper integral $\int_a^\infty \frac{dx}{x^p}$, $a>0$ is convergent at ∞, if $p>1$ and divergent at ∞, if $p\leq 1$.

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Section C

9. Write in short:

a) State mean value theorem for differentiable functions.

b) State the sufficient conditions for differentiability.

c) Evaluate $\int_{0}^{\infty} x^6 e^{-2x} dx$.

d) State improper integrals of first and second kinds.

e) Expand $x^2 y + 3y - 2$ in powers of $x-1$ and $y+2$ upto second degree terms.

f) State Jacobian of $n-$ functions.

g) State the sufficient conditions for the maxima and minima of a function of two variables.

h) Find the partial derivatives of first order when $z = \sin(x^2 y^2)$.

i) If $f(x, y, z) = 0$ find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

j) State Lebesgue's criterion for existence of a multiple Riemann integral.