10048/NJ

F-40/2110

ANALYSIS-I-301

SEMESTER-III

SYLLABUS-DECEMBER- 2019

TIME ALLOWED 3 Hrs M.M 70

NOTE: The candidates are required to attempt two questions each from Section A & B Section C will be compulsory

SECTION A

1. (a) Prove the every subset of a countable set is countable.

   (b) If $F = \{A_1, A_2, \ldots\}$ is a countable collection of sets, let $G = \{B_1, B_2, \ldots\}$ where $B_1 = A_1$ and, for $n > 1$, $B_n = A_n - \bigcup_{k=1}^{n-1} A_k$. Then prove that G is a collection of disjoint sets and $\bigcup_{k=1}^{\infty} A_k = \bigcup_{k=1}^{\infty} B_k$.

2. (a) If F is a countable collection of infinite disjoint sets such that each set is countable, then prove that their union is also countable.

   (b) Show that a set S in R^n is closed if, and only if, it contains all its adherent points.

3. State and Prove Bolzano-Weierstrass Theorem.

4. Let S be a subset of R^n. Then show that the following three statements are equivalent:

   (a) S is compact.

   (b) S is closed and bounded.

   (c) Every infinite subset of S has an accumulation point in S.

2x10= 20

SECTION B

5. (a) Suppose $Y \subset X$ then prove that a subset E of Y is open relative to Y if and only if $E = Y \cap G$ for some open subset G of X.

   (b) Prove that the compact subsets of metric spaces are closed.

6. (a) If (M,d) is a metric space, define $d'(x,y) = \dfrac{d(x,y)}{1+d(x,y)}$. Prove that d' is also a metric for M.

   (b) Prove that every compact subset of a metric space is complete.

7. Show that a contraction f of a complete metric space has a unique fixed point.

8. Prove that in a metric space (S,d) a sequence $\{x_n\}$ converges to p if, and only if, every subsequence $\{x_{k(n)}\}$ converges to p.

2x10= 20

SECTION C

9. Explain the following in short:

   a) Define compact and connected sets.

   b) Determine whether $d(x,y) = \sqrt{|x-y|}$ for $x, y \in R$ is a metric or not.

   c) State Cantor intersection and Lindelof covering theorems.

   d) Define countable and uncountable set with examples.

PTO

e) Show that arbitrary union of closed sets in a metric space need not be closed.

f) Determine all the accumulation points of "all the rational numbers" and decide whether the set is open or closed or neither.

g) In a metric space (S,d), assume that $x_n \to x$, $y_n \to y$. Prove that $d(x_n, y_n) \to d(x, y)$.

h) Let f be defined and continuous on a closed set S in R. Let $A = \{x : x \in S \ and \ f(x) = 0\}$. Prove that A is closed subset of R.

i) Give an example of a continuous $f : S \to T$ and a Cauchy sequence $\{x_n\}$ in some metric space S for which $\{f(x_n)\}$ is not a Cauchy sequence in T.

j) Define open set, closed set and accumulative point for a metric space (M,d).

$10 \times 3 = 30$