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707/MH

C-2050

DISCRETE MATHEMATICS-II

Paper-IV

Semester-VI

Time allowed : 2 Hours] [Maximum Marks : 40

Note : Attempt any four questions. All questions carry equal marks.

1. (i) Solve the recurrence relation $a_r = 2a_{r-1} + 3$, r 1. With $a_0 = 1$, by the method of generating function.

   (ii) Solve the following recurrence relation by Characteristic Roots Method

   $a_r = a_{r-1} + a_r - 2$, r 2. with $a_0 = 1, a_1 = 1$

2. (i) Discuss the Time Complexity of Algorithms.

   (ii) Determine the discrete numeric function corresponding to the following generating function is

   $$A(z) = \frac{z^5}{5 - 6z - z^2}$$

3. (i) Find the particular solution of the difference equation

   $a_r - 4a_{r-1} + 4a_{r-2} = (r)$, where $(r) = (r+1)2^r$

   (ii) What do you mean by Recursive Algorithm.

4. Design an algorithm to determine whether n colored balls placed in n boxes are of the same color. Basic operation is to compare two balls in any two boxes and find out whether they are of the same color or not. Determine the complexity of your algorithm in terms of the number of basic operations used.

5. (i) Prove that the following are equivalent in a Boolean Algebra.

   (i) $a + b = b$ (ii) a.b = a

   (iii) a' + b = 1 (iv) a.b' = 0

(ii) Show that the Boolean functions $_1 = (x_1 \quad x_2) \quad x_3$ and $_2 = x_1 \quad (x_2 \quad x_3)$ are equivalent.

6. (i) Prove that cube roots of unity form a fine abelian group under multiplication.

(ii) Prove that the set D_n of all positive divisors of n is a bounded distributive lattice.

7. (i) State and prove De-Morgan's Laws in Boolean Algebra.

(ii) Prove the validity of following arguments:

If man is a bachelor, he is unhappy.

If a man is unhappy, he dies young.

Therefore, bachelor die young.

8. (i) Let X be a non-empty set, then prove that poset ($P(X)$,) of all subsets of X is a lattice.

(ii) Give basic properties of Algebraic Systems defined by lattices.

9. (i) Define Generating Function of numeric functions.

(ii) What do you mean by Complexity of Problems.

(iii) Prove that the following are equivalent:

$p \quad q \quad (\sim q \quad \sim p)$

(iv) What do you mean by Recurrence relation.

(v) Define Boolean expression.

(vi) Prove that identity element of a Group is unique.

(vii) Find the general solution of recurrence relation

$S_n - 3S_{n-1} + 2S_{n-2} = 0$

(viii) Define Lattice.