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706/MH

C-2050

ALGEBRA-II

Paper-III

Semester - VI

Time Allowed : 2 Hours] [Maximum Marks : 40

Note : Attempt any four questions. All question carry equal marks.

1. Prove that a non empty subset W of a vector space V(F) is a subspace of V iff W is closed under addition and scalar multiplication.

2. Find the basis an dimension of the subspace W of R^4 generated by the following vectors : (1, 2, 3, 5); (2, 3, 5, 8); (3, 4, 7, 11); (1, 1, 2, 3). Also extend these to the basis of R^4.

3. If W is a subspace of a finite dimensional vector space V(F). Prove the $dim(V/W) = dim V - dim W$.

4. Prove that the necessary and sufficient conditions for a vector space $V(F)$ to be a direct sum of its subspaces W_1 and W_2 are

   (i) $V = W_1 + W_2$

   (ii) $W_1 \cap W_1 = \{0\}$

5. Prove that a linear tranformation T : V $\rightarrow$ W is non singular iff set of images of linearly independent set is linearly independent.

6. Let $T : V_3 \rightarrow V_3$ be defined as $T(x, y, z) = (3x, x - y, 2x + y + z)$. Show that T is invertible and T^{-1}.

7. Prove that the characteristics and minimal polynomials of an operator or a matrix have the same roots except for multiplicities.

8. Prove that the non-zero Eigne vector corresponding to distinct Elgen values of a linear operator are linearly independent.

9. Attempt all parts :

   (i) Prove that C is a vector space over field C.

   (ii) Let V be a vector space over the field F. Prove that every non-zero singleton subset of V is L.I. over F.

   (iii) If a basis of vector space V(F) contains n elements, Prove that a subset W of V having more than n elements of L.D.

   (iv) Define Quotient Space

   (v) Show that $T : R^2 \to R$ defined by $T(x, y) = xy$ is not a linear transformation.

   (vi) State Rank and Nullity theorem.

   (vii) If T is a linear operator on V such that $T^2 - T + 1 = 0$. Prove that T is invertible.

   (viii) Define Isomorphism.