

C-2050

## MATHEMATICAL METHODS-II

Option-iii

Semester-VI

Time allowed : 2 Hours] [Maximum Marks : 40

**Note :** Attempt any four questions. All questions carry equal marks.

1. (i) Find fourier sine integral of  $f(x)$  where

$$f(x) = \begin{cases} 2 - x & -1 < x < 1 \\ 0 & x < -1 \text{ or } x > 1 \end{cases}$$

- (ii) State and Prove Modulation theorem.

2. Find Fourier transform of

$$f(x) = \begin{cases} -|t| & - < t < \\ 0 & \text{otherwise} \end{cases}$$

where  $> 0$  and hence find  $\int_0^{\infty} \frac{\sin^2 t}{t^2} dt$ .

3. (i) Find finite Fourier sine and cosine transform of  $f(t) = 1$ .

(ii) Evaluate  $F^{-1}\left(\frac{e^{4is}}{3 + is}\right)$ .

4. Solve for  $f(t)$  if  $\int_0^t f(s) \sin st ds = \begin{cases} 1 & 0 < t < 1 \\ 2 - t & 1 < t < 2 \\ 0 & t > 2 \end{cases}$

Prove that  $f(t) = \frac{2}{t}(\cos t - 2 \cos 2t + 1)$ .

5. Solve  $\frac{d^2y}{dt^2} + y = 6 \sin 2t$  given  $y(0) = 2, y'(0) = 1$ .

6. Derive Wave Equation.

7. Solve  $\frac{dx}{dt} - 2y = t, \frac{dy}{dt} - 4x + 2y = 0$

given  $x(0) = 3, y(0) = 0$ .

8. Solve  $\frac{d^2y}{dt^2} + t \frac{dy}{dt} - y = 0;$

given  $y(0) = 0, y'(0) = 2$ .

9. (i) State Dirichlet's condition.
- (ii) If  $F(p)$  is the complex Fourier transforms of  $f(x)$ , then show that  $[f(x-a)] = e^{ipa} F(p)$ .
- (iii) State Linearity Property.
- (iv) Solve  $ty'' + y' + 4ty = 0$  if  $y(0) = 3, y'(0) = 0$ .
- (v) State shifting theorem.
- (vi) Show that  $xe^{\frac{-x^2}{2}}$  is self reciprocal with respect to Fourier sine transform.
- (vii) State Parseval's Identity.
- (viii) State Convolution Theorem.