PC-1696/M

L-4/2050

MATHEMATICAL METHODS-MM-716/AMC-425

(Semester–IV)

(Common for Math/AMC)

Time : Two Hours] [Maximum Marks : 70

Note : Attempt any *four* questions. All questions carry equal marks.

I. (a) Form an integral equation corresponding to the differential equation with the given conditions

$$y'' - 2xy = 0,\ y(0) = 1/2,\ y'(0) = y''(0) = 1.$$

(b) Find the solution of volterra integral equation of the second kind by the method successive substitutions.

II. (a) Solve by iterative method

$$y(x) = 1 + \lambda\int_0^{\pi} \sin(x+t)\,y(t)\,dt.$$

(b) Find the resolvent kernel for Volterra integral equation $K(x, t) = e^{x-t}$.

III. (a) Show that between Fredholm determinant $D(\lambda)$ and Fredholm's first minor $D(x, y; \lambda)$ the following relation holds

$$D(x, y; \lambda) - \lambda K(x, y)\, D(\lambda) = \lambda \int_a^b D(x,t;\lambda)\, k(t, y)\, dt$$

for all values of λ and for all x and y on rectangle R.

(b) Show that the series $D(x, y; \lambda)$ converges absolutely and uniformly in λ and on R : $a \le x \le b$ and $a \le y \le b$.

IV. State and prove Fredholm first fundamental theorem.

V. (a) Find the curve passing through (x_0, y_0) and (x_1, y_1) which generates the surface of minimum area when rotated about the x-axis.

(b) Find the general solution of Euler's equation corresponding to the functional

$$I(y) = \int_0^b f(x)\sqrt{1 + y'^2}\, dx \text{ where } f(x) = x.$$

VI. (a) Calculate the variational derivative at the point x_0 of the quadratic functional

$$J[y] = \int_a^b \int_a^b k(s,t)\; y(s)\, y(t)\, ds dt.$$

(b) Obtain a necessary condition for the curve $y_i = y_i(x)$ $(i = 1, 2, ... n)$ to be an extremal of the functional $\int_a^b f(x, y_1, y_2, ... y_n \;\; y_1', ...y_n')\, dx$ is that the functions $y_i(x)$ satisfy the Euler equation $F\, y_i - \frac{d}{dx} F\, y_i' = 0$.

$(i = 1, ... n)$

VII. (a) Determine the geodesic on the surface of a right circular cylinder.

(b) Show that a necessary and sufficient condition for the functional $\int_{t_0}^{f_1} \Phi\,(t, x, y, \dot{x}, \dot{y})\, dt$ to depend only on the curve in the xy-plane defined by $x = x(t)$, $y = y(t)$ and not on the choice of the parametric representation of the curve.

VIII. (a) Find an extremal in the following isoperimetric problem $J[y(x, \; z(x)] = \int_0^1 (y'^2 + z'^2 - 4xz - 4z)dx$, when $y(0) = z(0) = 0$, $y(1) = z(1) = 1$ and $\int_0^1 (y'^2 - xy'^2 - z'^2)dx = 2$.

(b) Determine the curve of length which passes through (0, 0) and (1, 0) and for which the area I between the curve and x-axis is maximum.

IX. (a) Define volterra integral equation of first kind.

(b) Define linear and non-linear integral equations.

(c) Define Fredholm integral equation of third kind.

(d) State Schwarz's inequality.

(e) State Neumann interior and exteria problem.

(f) Define functional.

(g) Solve $I(y) = \int_a^b F(x, y,)dx,$ where F does not dependent explicitly on y'.

(h) Define Geodesic.

(i) State Brachistochrone problem.

(j) State Fermat's principle.