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1003/MR

F-3/2050

LINER ALGEBRA

Paper–363

Semester–VI

Time Allowed : 2 Hours] [Maximum Marks : 45

Note : Attempt any **four** questions. All question carry equal marks.

1. Prove that the union of two subspaces is a subspace if and only if one of them is a subset of the other.

2. Examine whether the following set of vectors in $V_3(R)$ forms a basis or not :

   (1,0,–1), (1, 2, 1), (0, –3,2).

3. Find a linear transformation $T:P_3(x)\rightarrow P_3(x)$ such that $T(1+x)=1+x,\quad T(2+x)=x+3x^2, T(x^2)=0$, where $P_3(x)$ is the vector space of all polynomial of degree ≤ 3.

4. For the linear transformation $T:R^3\rightarrow R^3$ is defined by $T(x,y,z,)=(x+2y, y-z, x+2z)$, verify the Rank (T)+ Nullity (T)=3.

5. Let $T:V\rightarrow V$ be a linear operator, where V is a finite dimensional vector space over a field F. Suppose B is a basis of V(F). Prove that for any vector $v\in V,\quad [T;B][v;B]=[T(v);B]$.

6. If the matrix of a linear operator T or R^3 relative or usual basis is :

$$\begin{bmatrix} 1 & 1 & -1 \\ -1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix}$$

then find the matrix of T relative to the basis :

B={(1, 2, 2), (1, 1, 2), (1,2,1)}.

7. Let $T:V \to W$ be a linear transformation defined as $\{f(x)\} = \int_1^x f(t)\mathrm{dt}$, where $V = \{f(x) | f(x)$ is a polynomial over R and deg $f(x) \le 3$ or $f(x) = 0$. Let $B_1 = \{1, 1+x, 1-x+x^2\}$ and $B_2 = \{1, x, x^2, x^3\}$. Find the matrix representation of T relative to basis B_1 and $B_{2.}$

8. Find all the eigenvalues and eigenvectors of the matrix :

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}.$$

9. Write in brief :

(i) Check whether the system of vectors : u = (1, 2, –3, v =(1, –3, 2) and w =(2, –1,5) of V_3 (R) is linear Independent or not?

(ii) Prove that the polynomial $1, 1+x, 1+x^2, 1+x^3$ span the subspace W of polynomials in x of degree ≤ 3 (including zero polynomials over R).

(iii) Define singular and non-singular linear transformations.

(iv) Let $W = \{(a,b,c,d) | b-2c+d=0\}$ be a subspace of R^4. Find the dimension of W.

(v) Show that the map $T:R^2 \to R^3$ defined by $T(x,y) = (x+1, 2y-3, x-y)$ is not linear transformation.

(vi) Define annihilator and double annihilator.

(vii) Let V be a vector space over F. Prove that the set $\{v_1 v_2\}$ is linearly dependent if and only if v_1 and v_2 collinear.