PC-1002/MR

F-3/2050
MECHANICS-362

Time : Two Hours]   [Maximum Marks : 45

Note : Attempt any *four* questions. All questions carry equal marks.

I.   A uniform rod of length '*a*' hangs against a smooth vertical wall being supported by means of a string of length *l*, tied to one end of the rod, the other end of the string being attached to a point in the wall. Show that the rod can rest inclined to the wall at an angle given by:

$$\cos^2 \theta = \frac{l^2 - a^2}{3a^2}.$$

II.   Equal forces act along the coordinate axes and the line

$\dfrac{x - \alpha}{l} = \dfrac{y - \beta}{m} = \dfrac{z - \gamma}{n}$. Find the resultant wrench and the

equation of the central axes.

III. A system of forces given by (X, Y, Z, L, M, N) is replaced by two forces one of which acts along X-axis. Find the magnitude of the forces and line of action of the other force.

IV. A heavy uniform cube is balanced on the highest point of a sphere whose radius is *a*. If the sphere is rough enough to prevent sliding and if the side of the cube be $\dfrac{\pi a}{2}$, show that a cube can rock through a right angle without falling.

V. Find expressions for velocity along and perpendicular to the radius vector of a particle moving along a plane curve.

VI. A particle move with S.H.M. in a straight line. In the first second after starting from rest it travels a distance '*a*' and in the next second, it travels a distance '*b*'. Prove that the amplitude of the motion is $\dfrac{2a^2}{3a - b}$.

VII. A particle describes the path $r = a \tan \theta$ under a force to the origin. Find the acceleration and velocity in terms of *r*.

VIII. A particle thrown over an isosceles right angled ΔABC, right angled at B from one end A of the horizontal base and grazing the vertex B falls at C. Show that the angle of projection is $\tan^{-1} 2$.

IX. All Questions are Compulsory.

(a) Prove that Simple Harmonic Motion is periodic with period $\dfrac{2\pi}{\sqrt{\mu}}$.

(b) Define power and energy.

(c) What is the equation of null plane?

(d) What is the expression for acceleration perpendicular to the radius vector of a particle moving along a plane curve?

(e) Define Angular momentum.

(f) Prove that the rate of change of direction of velocity of a particle moving in cycloid is constant.

(g) What are Kepler's Law of Motion?

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