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421/MH

C-2050

ALGEBRA-II

Option-III

Semester-VI

Time allowed : 2 Hours] [Maximum Marks : 40

Note : Attempt any four questions. All questions carry equal marks.

1. If u, v, w are linearly independent vectors in a vector space $V(F)$, Then show that

   (i) The vectors $u + v$, $v + w$, $w + u$ are linearly independent.

   (ii) The vectors $u + v$, $u - v$, $u - 2v + w$ are linearly independent.

2. If w is a subspace of a finite dimensional vector space V(F), prove that dim W   dimV. Measure W = V iff dim W = dimV.

3. (i) If W_1 is a vector space at V(F), show that there exists, a subspace W_2 of V(E) such that $V = W_1 \quad W_2$.

   (ii) For a subset S of vector space V(F), show that the linear span of S is the smallest subspace of V(F).

4. Find a basis and dimension of the solution space of system of homogeneous linear equation :

$$x + 2y - 4z + 3s - t = 0$$

$$x + 2y - 2z + 2s + t = 0$$

$$2x + 4y - 2z + 3s + t = 0$$

5. (i) Let $T : R^3 \quad R^3$ be defined by $T(x, y, z) = (3x, x - y, 2x + y + z)$. Prove that $(T^2 - I)(T - 3I) = 0$.

   (ii) Prove that a linear transformation T : V   W is non-singular iff T is one-one.

6. Prove that the characteristic and minimal polynomials of an operator have the same roots except for multiplicities.

7. Let T be a linear operator defined by

$T(x, y, z) = (2y + z, x - 4y, 3x)$

(i) Find the matrix of T relative to basis B = {(1, 1, 1,), (1, 1, 0), (1, 0, 0)}

(ii) Verify that [T ; B] [v ; B] = [T(v) ; B] v R^3.

8. Find all eigen values and basis for each eigen space of linear operator T : R^3 R^3 defined by $T(x, y, z) = (3x + y + 4z, 2y + 6z, 5z)$.

9. (i) Show that W = {(a, b, 0)} in R^3 is generated by $u = (1, 2, 0)$ and $v = (0, 1, 0)$.

(ii) For what value of K, the vector (1, K, 5) is a linear combination of (1, –3, 2) and (2, –1, 1) in R^3.

(iii) Prove that e is a vector space over IR.

(iv) Let T : V   W is a linear transformation. Prove that null space of T is a subspace of V(F).

(v) Define isomorphic vector spaces.

(vi) If T is a linear operator on V such that $T^2 - T + I = 0$. Prove that T is invertible.

(vii) What are isomorphic vector spaces.

(viii) Find the characteristic and minimal polynomial for linear operator T : R^2 R^2 defined by $T(x, y) = (x, y, 2x)$.