PC-3871/MR

G-9/2050

NUMBER THEORY–601

(Semester–VI)

Time : Two Hours] [Maximum Marks : 70

Note : Attempt any *four* questions. All questions carry equal marks.

I. If A_n is the nth prime number then Prove that $A_n \leq 2^{2^{n-1}}$.

II. State and Prove Fundamental theorem of arithmetic.

III. (a) For arbitrary integers a and b, $a = b \pmod{n}$ if and only if a and b leave the same non-negative remainder when divided by n.

(b) By using the definition of Congruence show that 41 divides $2^{20} - 1$.

IV. (a) State and prove Mobius Inversion Formula.

(b) Prove that $\mu(n)$ is a multiplicative function.

V. Define Quadratic reciprocity law. Using the Generalized Quadratic Reciprocity Law, determine whether the congruence $x^2 \equiv 231 \pmod{1105}$ is solvable.

VI. (a) Prove that $ax + by = a + c$ is solvable iff $ax + by = c$ is solvable.

(b) Find all the solutions of $10x - 7y = 17$.

VII. Define Pell's equation. Prove that if d is a positive integer not a perfect square, then $h_n^2 - dk_n^2 = (-1)^{n-1} q_{n-1}$ for all the integers $n \geq -1$.

VIII. Prove that the continued fraction expansion of the real quadratic irrational number 'a' is purly periodic iff $a > 1$ and $-1 < a* < 0$, where a^* is the conjugate of a.

IX. (a) Show that $M(a, b) = M(a \pm b, b)$.

(b) Find the index of 5 relative to each of the primitive roots of 13.

(c) Show that 125671221 is divisible by 9.

(d) Find the remainder when 2(28!) is divided by 31.

(e) State Chinese remainder theorem.

(f) Find the solution of $x^2 \equiv 5 \pmod{29}$.

(g) Find all the quadratic residue of 13.

(h) Evaluate n of Gauss Lemma for (5/19).

(i) Find the value of Jacobi Symbol $\left(\frac{22}{105}\right)$.

(j) Define continued fraction and give example.